MATH 506 Ergodic Theory Spring 2023 Schedule

Lec.	Date	Topic	Sections
1	1/9	Introduction. Measure spaces – review.	0.2
2	1/11	Measure-preserving transformations: basic properties and examples.	1.1
3	1/13	The Haar measure on compact metrizable topological groups.	
-	1/16	*Martin Luther King Day – no classes.*	
4	1/18	Endomorphisms of the circle and the tori.	0.8
5	1/20	Product spaces, the product measure, and the shift map.	0.2, 1.1
6	1/23	Poincaré's Recurrence Theorem. Ergodicity in terms of sets.	1.4, 1.5
7	1/25	Integration and L^p spaces – review. Induced operator on function spaces.	0.3, 0.5, 1.3
8	1/27	Ergodicity in terms of invariant functions.	1.5
9	1/30	Discussion. Ergodicity of rotations of circles and tori.	1.5
10	2/1	Characters. Ergodicity of translations on compact topological groups.	0.7, 1.5
11	2/3	Ergodicity of group endomorphisms.	1.5
12	1/6	Ergodicity of shifts. Birkhoff Ergodic Theorem and corollaries.	1.5, 1.6
13	2/8	Conditional expectations. A proof of Birkhoff Ergodic Theorem.	0.4, 1.6
14	2/10	L^p Ergodic Theorem. Normal numbers.	1.6
15	2/13	Ergodicity, weak mixing, and mixing.	1.6, 1.7
16	2/15	Weak mixing.	1.7
17	2/17	Ergodicity, weak mixing, and mixing of products.	1.7
18	2/20	Ergodicity, weak mixing, and mixing in functional form.	1.7
19	2/22	Ergodicity, weak mixing, and spectral properties.	1.7
20	2/24	Stochastic matrices and Markov shifts.	1.1
21	2/27	Some properties of stochastic matrices and Markov shifts.	1.7
22	3/1	Ergodicity of Markov shifts.	1.7
23	3/3	Mixing properties of Markov shifts.	1.7
		3/5 - 3/11 Spring break – no classes.	
24	3/13	Notions of equivalence for probability spaces.	2.1
25	3/15	Conjugacy and L^2 spaces. Isomorphism of measure-preserving transformations.	2.1, 2.2
26	3/17	Conjugacy and spectral isomorphism of measure-preserving transformations.	2.3, 2.5
27	3/20	Spectral invariants. Countable Lebesgue spectrum.	2.5
28	3/22	Eigenvalues and eigenfunctions. Discrete spectrum.	3.1, 3.2
29	3/24	Discrete spectrum. Representation Theorem.	3.2, 3.3
30	3/27	Partitions and subalgebras. Entropy of a partition.	4.1, 4.2
31	3/29	Conditional entropy.	4.3
32	3/31	Conditional entropy. Entropy of a measure-preserving transformation.	4.3, 4.4
33	4/3	Properties of the entropy of a measure-preserving transformation.	4.4, 4.5
34	4/5	Zero entropy.	4.5
35	4/7	Conditional entropy given arbitrary sub-σ-algebra. Kolmogorov-Sinai Theorem.	4.3, 4.6
36	4/10	Entropy of Bernoulli and Markov shifts.	4.7, 4.9
37	4/12	Entropy of group rotations and entropy of products.	4.6, 4.7
38	4/14	Entropy of circle endomorphisms and toral automorphisms.	notes
39	4/17	Entropy and Lyapunov exponents – an overview.	notes
40	4/19	Measures on metric spaces. Weak*-topology.	6.1
41	4/21	Invariant measures for continuous transformations.	6.2
42	4/24	Ergodic measures for continuous transformations.	6.2
43	4/26	Examples. Unique ergodicity.	6.5, 6.6
44	4/28	More on invariant measures. Variational Principle. Measures of max entropy.	6.5, 8.2, 8.3